First Round: February 29, 2020 at Regional Testing Centers
Second Round: April 25, 2020 at Auburn University at Montgomery

GEOMETRY EXAMINATION

Construction of this test directed
by
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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees.
- If A and B are points on a circle, then $\widehat{AB}$ is the arc between A and B.

- If A and B are points on a circle, then $m\widehat{AB}$ is the measure of $\widehat{AB}$ in degrees.
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If $\triangle ABC \cong \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are congruent.
- If $\triangle ABC \sim \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are similar.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer

- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining

- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems.

1. The measures of the angles of a triangle are in the ratio of 1:7:4. Which of the following is the measure of one of the angles?

   (**A**) 90°   (**B**) $\boxed{105°}$ (**C**) 120°   (**D**) 135°   (**E**) None of these

2. The square $DEFG$ has its vertices on the three sides of right triangle $\triangle ABC$ as shown in the figure. If the area of $DEFG$ is 36, and $BD = 3$, find the area of $\triangle ABC$.
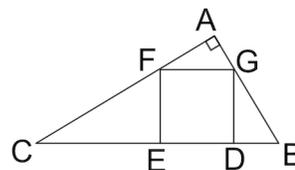
   (**A**) $\dfrac{63\sqrt{5}}{5}$ (**B**) $\dfrac{108\sqrt{5}}{5}$ (**C**) $\dfrac{324}{5}$ (**D**) $\boxed{\dfrac{441}{5}}$ (**E**) None of these

3. If the radius of a circle is increased by 1 unit, what is the ratio of the new circumference to the new diameter?

   (**A**) $\dfrac{\pi}{2}$ (**B**) $\pi + 1$ (**C**) $\boxed{\pi}$ (**D**) 2π (**E**) None of these

4. Find the equation of the line that is the perpendicular bisector of the line segment with end points $(8, 14)$ and $(2, 6)$.

   (**A**) $3x - 4y = -35$ (**B**) $4x - 3y = -10$

   (**C**) $3x + 4y = 50$ (**D**) $\boxed{3x + 4y = 55}$ (**E**) None of these

5. Let $\triangle ABC$ be a right triangle with right angle at vertex C. A second right triangle, $\triangle ABD$ is also constructed with hypotenuse $\overline{AB}$ so that point D exterior to triangle $\triangle ABC$. If $BC = 1$, $AC = 3$ and $AD = 2$, find BD.

   (**A**) $\boxed{\sqrt{6}}$ (**B**) $\sqrt{10}$ (**C**) 2   (**D**) 4   (**E**) None of these

6. Three vertices of parallelogram $PQRS$ are $P(-3, -2)$, $Q(1, -5)$ and $R(9, 1)$, with P and R diagonally opposite. What is the sum of the coordinates of vertex S?

   (**A**) 7   (**B**) 8   (**C**) $\boxed{9}$ (**D**) 10   (**E**) None of these

7. What is the largest area of a triangle that can be inscribed in a semicircle of radius r?

   (**A**) $\dfrac{r^2}{4}$ (**B**) $\boxed{r^2}$ (**C**) $\sqrt{2}\,r^2$ (**D**) $2r^2$ (**E**) None of these

8. Points P, S, and T all lie on the circle pictured, with $m\angle TPS = 36°$. What is the length of minor arc $\widehat{ST}$ if the radius of the circle is 15 inches?

   (**A**) 3π (**B**) 4π (**C**) 5π (**D**) $\boxed{6\pi}$ (**E**) None of these
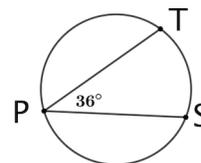
9. If the graphs of $3y + kx = -2$ and $2y + x = -3$ are to meet at right angles, what is the value of k?

   (**A**) $-\dfrac{1}{6}$ (**B**) $-\dfrac{2}{3}$ (**C**) $\dfrac{1}{6}$ (**D**) $\dfrac{3}{2}$ (**E**) $\boxed{\text{None of these}}$

10. In triangle $\triangle ABC$, $AB = 16$, $AC = 9$ and the median from vertex C has length 11. Find the perimeter of $\triangle ABC$.

(**A**) 36   (**B**) $\boxed{42}$ (**C**) 50   (**D**) 53   (**E**) None of these

11. In the figure shown, points D, B, E and C are on the circle, A, B, and D are colinear, A, C, and E are colinear, $m\angle DAE = 40°$, and $m\widehat{BD} = m\widehat{DE} = m\widehat{CE}$. Determine the measure of $\angle DBE$.
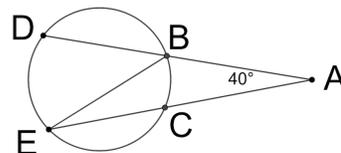
(**A**) $\boxed{55°}$ (**B**) $60°$ (**C**) $65°$ (**D**) $70°$ (**E**) None of these

12. A regular n-gon has interior angles of measure $176°$. How many sides does this n-gon have?

(**A**) 72   (**B**) 78   (**C**) 84   (**D**) $\boxed{90}$ (**E**) None of these

13. Find the x-coordinate of the point on the x-axis that is equidistant to $(1, -1)$ and $(-5, 5)$.

(**A**) $\boxed{-4}$ (**B**) -3 (**C**) -2 (**D**) 0   (**E**) None of these

14. The perimeter of a particular square and equilateral triangle are equal. If the height of the equilateral triangle is $2\sqrt{3}$, what is the positive difference between the area of the square and the area of the triangle?

(**A**) $\dfrac{9 - 4\sqrt{3}}{4}$ (**B**) $\dfrac{3 - \sqrt{3}}{2}$ (**C**) $\boxed{9 - 4\sqrt{3}}$ (**D**) $6 - 2\sqrt{3}$ (**E**) None of these

15. A rectangular field is half as wide as it is long, and is completely enclosed by x yards of fencing. The area of the field in terms of x is

(**A**) $\boxed{\dfrac{x^2}{18}}$ (**B**) $\dfrac{x^2}{9}$ (**C**) $\dfrac{2x^2}{9}$ (**D**) $\dfrac{x^2}{3}$ (**E**) None of these

**16.** A chord of length $16\sqrt{3}$ is the perpendicular bisector of the radius in a circle. Determine the area of the circle.

(**A**) 192π (**B**) $\boxed{256\pi}$ (**C**) 324π (**D**) 384π (**E**) None of these

17. What is the measure of an angle whose supplement is 400% of its complement?

(**A**) $20°$ (**B**) $30°$ (**C**) $40°$ (**D**) $\boxed{60°}$ (**E**) None of these

18. A parallelogram $ABCD$ has side $AB = 12$ and diagonal $BD = 12$. Find the area of the parallelogram.

(**A**) $48\sqrt{3}$ (**B**) $72\sqrt{3}$ (**C**) $36\sqrt{5}$ (**D**) $18\sqrt{15}$ (**E**) $\boxed{\text{Not enough information}}$

19. In right triangle $\triangle ABC$, the point D on $\overline{AB}$ has $AD = 8$, $m\angle CDB = 60°$, and $m\angle CAB = 30°$. Find BC.

(**A**) $\boxed{4\sqrt{3}}$ (**B**) $4\sqrt{5}$ (**C**) $6\sqrt{3}$ (**D**) $6\sqrt{5}$ (**E**) None of these
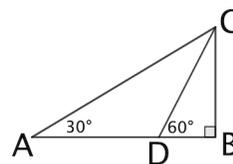
20. In a particular circle, the difference between the area of the circumscribed triangle and the inscribed triangle is 25 sq. in. If both triangles are regular polygons, find the radius of the circle.

(**A**) $\frac{7}{3}\sqrt{\frac{\sqrt{3}}{3}}$ (**B**) $\boxed{\frac{10}{3}\sqrt{\frac{\sqrt{3}}{3}}}$ (**C**) $\frac{11}{3}\sqrt{\frac{\sqrt{3}}{3}}$ (**D**) $\frac{7}{5}\sqrt{\frac{\sqrt{5}}{5}}$ (**E**) $\frac{11}{5}\sqrt{\frac{\sqrt{5}}{5}}$

21. The interior angle measures in degrees of a pentagon are $2x + 30$, $2x + 50$, $2x + 70$, $2x + 90$, and $2x + 110$. Find the measure of the largest angle, in degrees.

(**A**) 108   (**B**) 132   (**C**) 140   (**D**) $\boxed{148}$ (**E**) None of these

22. In the triangle pictured, angles $\angle ABC$, $\angle BDC$ and $\angle AED$ are all right angles, point D is on $\overline{AC}$, point E is on $\overline{AB}$, and $m\angle CAB = 20°$. What is the measure of $\angle BDE$?

(**A**) $\boxed{20°}$ (**B**) $30°$ (**C**) $60°$ (**D**) $70°$ (**E**) None of these
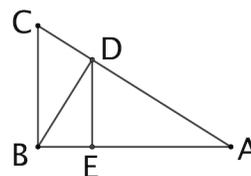
23. For what range of values of k do the lines $kx + y = 3$ and $x - y = 2$ intersect in the first quadrant?

(**A**) $k < \frac{1}{2}$ (**B**) $2 < k$ (**C**) $\boxed{-1 < k < \frac{3}{2}}$ (**D**) $2 < k < 3$ (**E**) None of these

24. A given cylindrical can is made up of a square of metal and two circular disks, each with a diameter of k cm. What is the volume of this can?

(**A**) $\dfrac{\pi^2 k^3}{2}$ (**B**) $\boxed{\dfrac{\pi^2 k^3}{4}}$ (**C**) $\dfrac{\pi^2 k^3}{6}$ (**D**) $\pi^2 k^3$ (**E**) None of these

25. When the base of a triangle is increased by 10%, and the altitude to that base is decreased by 10%, what is the effect on the area?

(**A**) $\boxed{\text{1\% decrease}}$ (**B**) 0.5% decrease   (**C**) No change   (**D**) 0.5% increase   (**E**) 1% increase

26. In the diagram shown, $\overline{XW}$ and $\overline{YZ}$ are common, external tangents to equal, tangent circles centered at O and P. If $XW = 6$, find the area of the shaded region.

(**A**) $36 - 8\pi$ (**B**) $\boxed{36 - 9\pi}$ (**C**) $72 - 36\pi$

(**D**) $72 - 9\pi$ (**E**) None of these
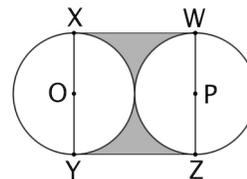
27. The circumference of a circle is 100 cm. What is the perimeter of a square inscribed in this circle?

(**A**) $\dfrac{50\sqrt{2}}{\pi}$ (**B**) $\dfrac{100\sqrt{2}}{\pi}$ (**C**) $\boxed{\dfrac{200\sqrt{2}}{\pi}}$ (**D**) $\dfrac{300\sqrt{2}}{\pi}$ (**E**) None of these

28. A goat is tied to a stake in the middle of a pasture with a 14 foot rope. If the goat eats 77 square feet of grass per day, for approximately how many days will the goat have enough grass to eat?

(**A**) 4   (**B**) 5   (**C**) 6   (**D**) 7   (**E**) $\boxed{8}$

29. Three distinct lines L_1, L_2 and L_3 lie in a plane so that L_2 intersects L_1 and L_3 is parallel to L_1. How many points in the plane are equidistant to all three lines?

(**A**) 0   (**B**) 1   (**C**) $\boxed{2}$ (**D**) 3   (**E**) None of these

30. The sum of the roots of the equation $4 - x = \dfrac{3}{x}$ is the radius of a sphere. Find the surface area of the sphere.

(**A**) $\dfrac{64\pi}{3}$ (**B**) $\boxed{64\pi}$ (**C**) $\dfrac{256\pi}{3}$ (**D**) 256π (**E**) None of these

31. A right rectangular prism has surface area of 1000 square inches, and has a base width and length of 10 inches and 20 inches, respectively. Find the volume of the prism in cubic inches.

(**A**) 1500   (**B**) $\boxed{2000}$ (**C**) 3000   (**D**) 4000   (**E**) None of these

32. In the figure, point B is the midpoint of $\overline{AC}$, and point D is placed so that $DA = DB$, and $DB = BC = 10$. Find DC.

(**A**) $10\sqrt{2}$ (**B**) $\boxed{10\sqrt{3}}$ (**C**) $20\sqrt{2}$

(**D**) $20\sqrt{3}$ (**E**) None of these
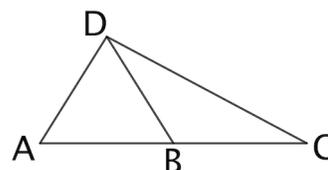
33. If the measures of an angle and its complement are in the ratio 2:7, what is the measure of the supplement of the angle, in degrees?

(**A**) 50   (**B**) 70   (**C**) $\boxed{160}$ (**D**) 170   (**E**) None of these

34. A five pointed star is inscribed into a circle, as shown in the picture. Find $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E$.

(**A**) $150°$ (**B**) $175°$ (**C**) $\boxed{180°}$ (**D**) $225°$ (**E**) None of these
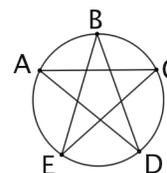
35. A 1 cm wide border is removed from a rectangular sheet of cardboard, resulting in a rectangle whose area is half that of the original rectangle. If the original perimeter was 28 cm, find the original area in square cm.

(**A**) 33   (**B**) 40   (**C**) 45   (**D**) $\boxed{48}$ (**E**) None of these

36. Find the area of a trapezoid in which the bases measure 17 and 42, and the legs measure 15 and 20.

(**A**) 94   (**B**) 150   (**C**) $\boxed{354}$ (**D**) 630   (**E**) None of these

37. What is the length of the hypotenuse of a right triangle that has an area of 40 and a longest leg of length 16?

(**A**) $\sqrt{81}$ (**B**) $2\sqrt{41}$ (**C**) $\sqrt{137}$ (**D**) $\boxed{\sqrt{281}}$ (**E**) None of these

38. In what type of triangle do the incenter and centroid coincide?
I. Equilateral Triangles   II. Isosceles, Right Triangles   III. Isosceles, Obtuse Triangles

(**A**) I only   (**B**) II   (**C**) II and III   (**D**) I and II   (**E**) I, II, and III

39. Triangle $\triangle ABC$ is isosceles with base $\overline{AC}$. Points P and Q are on $\overline{BC}$ and $\overline{AB}$, respectively, so that $AC = AP = PQ = QB$. Find the measure of angle $\angle ABC$, in degrees.

(**A**) 18   (**B**) 20   (**C**) $\dfrac{45}{2}$ (**D**) $\dfrac{180}{7}$ (**E**) None of these

40. In Trapezoid $DCBA$, $\overline{CD}$ is parallel to $\overline{AB}$, and $\overline{AD}$ is perpendicular to $\overline{AB}$. Let P be a point on $\overline{AB}$ so that the area of $APCD$ equals the area of $\triangle PBC$. If $CD = 1$, $AD = 4$, $AB = 10$, find PB.

(**A**) 2.5   (**B**) 3   (**C**) 4   (**D**) 5.5   (**E**) None of these

41. A square whose area is 64 is partitioned into four congruent squares, as shown. Find the circumference of the circle that passes through the centers of each of the four smaller squares.

(**A**) $2\sqrt{2}\,\pi$ (**B**) $4\sqrt{2}\,\pi$ (**C**) 4π (**D**) 8π (**E**) None of these
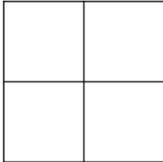
42. A rectangle is three times as long as it is wide. If it has a diagonal of length 10, what is the area of the rectangle?

(**A**) 10   (**B**) 20   (**C**) 30   (**D**) 40   (**E**) None of these

43. Find the center (h, k) of the circle given by the equation $x^2 + y^2 - 4x - 6y - 23 = 0$.

(**A**) $(-16, -36)$ (**B**) $(-2, -3)$ (**C**) $(2, 3)$ (**D**) $(16, 36)$ (**E**) None of these

44. Find the volume of a cube with a surface area of 150 square units.

(**A**) $64u^3$ (**B**) $125u^3$ (**C**) $216u^3$ (**D**) $225u^3$ (**E**) None of these

45. A bug that cannot fly is sitting in one corner of a 9 foot by 12 foot room, with a 7 foot ceiling. In the extreme opposite corner, the bug sees a crumb. What is the length of the shortest path from the bug to the crumb, in feet?

(**A**) $5\sqrt{3}$ (**B**) $8\sqrt{3}$ (**C**) 15   (**D**) 20   (**E**) None of these

46. In triangle ABC, $AB = 3$, $BC = 4$ and $AC = 6$. If $\overline{BC}$ is extended past C to point D so that $CD = BC$, find AD.

(**A**) $2\sqrt{13}$ (**B**) $3\sqrt{17}$ (**C**) $\sqrt{91}$ (**D**) $\sqrt{95}$ (**E**) None of these

47. Triangle $\triangle ABC$ is formed by the vertices $A(8, 3)$, $B(4, 1)$ and $C(-5, 4)$. Determine the length of the altitude to side $\overline{AB}$.

(**A**) $3\sqrt{5}$ (**B**) $5\sqrt{5}$ (**C**) $7\sqrt{5}$ (**D**) $9\sqrt{5}$ (**E**) None of these

48. The radius of the circle shown is 9 feet 4 inches, and arc $\overarc{AB}$ has length 5 feet 10 inches. Determine the measure of angle $\angle AOB$ in degrees.

(**A**) $\dfrac{128}{\pi}$ (**B**) $\boxed{\dfrac{225}{2\pi}}$ (**C**) $\dfrac{175}{2}\pi$ (**D**) 110π (**E**) None of these
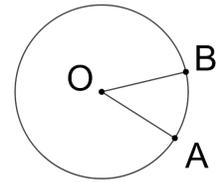
49. Find the area of the circle passing through the three points $(-6, 5)$, $(-3, -4)$, and $(2, 1)$.

(**A**) $\boxed{25\pi}$ (**B**) 30π (**C**) 40π (**D**) 45π (**E**) None of these

50. Let x be the measure of one angle of a regular pentagon, and y be the measure of one angle of a regular octagon. Find $x + y$.

(**A**) $228°$ (**B**) $\boxed{243°}$ (**C**) $248°$ (**D**) $255°$ (**E**) None of these