$\mathcal{ALABAMA}$

STATEWIDE MATHEMATICS CONTEST

First Round: February 22, 2014 at Regional Testing Centers
Second Round: March 15, 2014 at The University of North Alabama

GEOMETRY EXAMINATION

Construction of this test directed
by
Scott H. Brown, Auburn University Montgomery

INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B

- If A is an angle, then:
  $m\angle A$ is the measure of angle A in degrees

- If A and B are points on a circle, then:
  $\overarc{AB}$ is the arc between A and B $m\overarc{AB}$ is the measure of $\overarc{AB}$ in degrees

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Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer

- Cryptoanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining

- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

The following information is courtesy of the U.S. Bureau of Labor Statistics.

- The median salary of a Mathematician in 2012 was $101,360 per year.
- Over the next 10 years, the job opportunities for mathematicians are expected to grow by 23%!

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of North Alabama offers an undergraduate degree in Mathematics and has many great things to offer, including a new Mathematics Fellow program, an active undergraduate research group and a new Dual Degree Engineering program. For more information, go to www.una.edu/math.

1. An isosceles triangle has two sides of length 10 and one of length 12. What is its area?

   (**A**) 36 (**B**) 48 (**C**) 52 (**D**) 64 (**E**) 72

2. A regular polygon has interior angles of 168°. How many sides does the polygon have?

   (**A**) 20 (**B**) 25 (**C**) 30 (**D**) 35 (**E**) 40

3. The interior angles of a pentagon are $2x$, $2x+40$, $2x+80$, $2x+120$ and $2x+160$. What is the measure of the smallest angle?

   (**A**) 28° (**B**) 16° (**C**) 32° (**D**) 48° (**E**) 12°

4. Segment $\overline{OB}$ bisects angle $\angle AOC$. If $m\angle AOB = 2x + 10$ and $m\angle BOC = 8x - 14$, find $m\angle AOC$.

   (**A**) 24° (**B**) 28° (**C**) 32° (**D**) 34° (**E**) 36°

5. An octahedron has how many edges?

   (**A**) 8 (**B**) 30 (**C**) 6 (**D**) 12 (**E**) 20

6. In the given figure, $CD = BE$ and $\angle BDE = \angle BAC$. Find the perimeter of the triangle ABC.
   Problem thrown out. There is not a consistent scale factor for the triangles, so with the numbers provided they are not actually similar.

   (**A**) 135 (**B**) 144 (**C**) 152 (**D**) 160 (**E**) 162
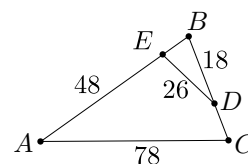
7. The minute hand of a town clock measures 12 feet. How far, in feet, does the tip of the minute hand travel in 35 minutes?

   (**A**) 14π (**B**) 12π (**C**) 10π (**D**) 16π (**E**) 13π

8. What is the midpoint of the line segment connecting the points $(2, -8)$ and $(-6, 5)$?

   (**A**) $(-2, 1)$ (**B**) $(2, \frac{3}{2})$ (**C**) $(2, 1)$ (**D**) $(-2, -\frac{3}{2})$ (**E**) $(-1, -\frac{1}{2})$

9. Find the surface area of a rectangular solid with a length of 7 inches, a width of 3 inches and a height of 5 inches.

   (**A**) 108 in^2 (**B**) 142 in^2 (**C**) 126 in^2 (**D**) 166 in^2 (**E**) 138 in^2

10. A cube with edges of length 6 is inscribed in a sphere. Determine the volume of the sphere.
    Problem thrown out. The $\sqrt{2}$ below should be $\sqrt{3}$. Then the answer is B.

    (**A**) $144\sqrt{2}\pi$ (**B**) $108\sqrt{2}\pi$ (**C**) $96\sqrt{2}\pi$ (**D**) $72\sqrt{2}\pi$ (**E**) $36\sqrt{2}\pi$
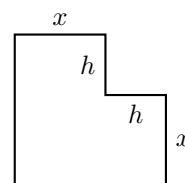
    Figure is in 3-Dimensions
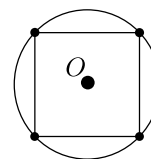
    Side View

11. If the radius of a circle is increased by 100%, the area is increased by:

    (**A**) 100% (**B**) 200% (**C**) 300% (**D**) 400% (**E**) 800%

12. In the given figure, all adjacent sides meet at right angles. If the area of the figure is $60u^2$ and if $3 < x < 5$, then:

(**A**) $\boxed{3.5 < h < 8.5}$ (**B**) $0.5 < h < 3.5$

(**C**) $1.5 < h < 5.5$ (**D**) $6 < h < 10$ (**E**) $2.5 < h < 6.5$

13. Which equation is a line perpendicular to $x + 2y = 8$?

(**A**) $y = \frac{1}{2}x + 4$ (**B**) $y = \frac{1}{2}x + 8$ (**C**) $y = -2x + 8$ (**D**) $y = -\frac{1}{2} + 4$ (**E**) $\boxed{y = 2x + 8}$

14. The sides of a triangle are 14, 16 and 18. Determine the length of the shortest altitude.

(**A**) $\frac{24\sqrt{5}}{9}$ (**B**) $4\sqrt{5}$ (**C**) $\boxed{\frac{48\sqrt{5}}{9}}$ (**D**) $8\sqrt{5}$ (**E**) $\frac{64\sqrt{5}}{9}$

15. Given two similar triangles, the area of the larger triangle is sixteen times the area of the smaller triangle. What is the ratio of the perimeter of the larger triangle to the perimeter of the smaller triangle?
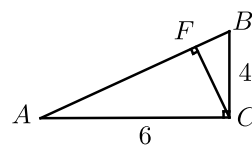
(**A**) $\boxed{4 : 1}$ (**B**) $\sqrt{8} : 1$ (**C**) $16 : 1$ (**D**) $32 : 1$ (**E**) $8 : 1$

16. In a triangle, one angle is three times as large as the other and the third is $20°$ greater than the sum of the other two. What are the measures of the angles of the triangle?

(**A**) $5°, 15°, 160°$ (**B**) $10°, 30°, 140°$ (**C**) $\boxed{20°, 60°, 100°}$ (**D**) $25°, 75°, 80°$ (**E**) $30°, 60°, 90°$

17. In the right triangle ABC pictured below, $\overline{CF}$ is perpendicular to $\overline{AB}$. Find the length of $\overline{CF}$.

(**A**) $\frac{\sqrt{13}}{13}$ (**B**) $\frac{2\sqrt{13}}{13}$ (**C**) $\frac{6\sqrt{13}}{13}$ (**D**) $\boxed{\frac{12\sqrt{13}}{13}}$ (**E**) $\sqrt{13}$

18. What is the area of a circle inscribed in a dodecagon with an apothum of length 13?

(**A**) 72π (**B**) $\boxed{169\pi}$ (**C**) 81π (**D**) 121π (**E**) 144π

19. If a rectangle has diagonal length c and perimeter p, which of the following is an expression for the area of the rectangle?

(**A**) $\boxed{\frac{p^2 - 4c^2}{8}}$ (**B**) pc (**C**) $\frac{\sqrt{p^2 + 2c^2}}{2}$ (**D**) $p^2 + c^2$ (**E**) $\frac{p^2 + c^2}{2}$

20. A circle of radius 20 inches has its radius reduced by 4 inches. By what percentage has its area been decreased?

(**A**) 81% (**B**) 44% (**C**) 50% (**D**) 64% (**E**) $\boxed{36\%}$

21. The perimeter of a right triangle is 42 and the sum of the squares of its sides is 722. Determine the area of the right triangle.

(**A**) 100 (**B**) 64 (**C**) 81 (**D**) $\boxed{42}$ (**E**) 121

22. The angle of a sector of a circle is $96°$ and its corresponding arc is 24 inches. Find the area of the circle.

(A) $\dfrac{1950}{\pi}$ (B) $\dfrac{1800}{\pi}$ (C) $\boxed{\dfrac{2025}{\pi}}$ (D) $\dfrac{2175}{\pi}$ (E) $\dfrac{1845}{\pi}$

23. Find the area of a trapezoid in which the bases are 17 feet and 42 feet and the legs are 15 feet and 20 feet.

(A) 340 ft^2 (B) $\boxed{354 \text{ ft}^2}$ (C) 455 ft^2 (D) 595 ft^2 (E) 714 ft^2

24. Debbie bought a box in the shape of a rectangular prism. She finds the lengths of the diagonals of the faces to be 20, $2\sqrt{61}$, and $2\sqrt{89}$. Determine the volume of the box.

(A) 2020 u^2 (B) 1840 u^2 (C) 1960 u^2 (D) $\boxed{1920 \text{ u}^2}$ (E) 2200 u^2

25. In the figure shown, $\overline{AB}$ is parallel to $\overline{CD}$ and the distance between them is 8. What is the length of the radius of the circle?

(A) 8 (B) $\boxed{\sqrt{85}}$ (C) 9 (D) $\sqrt{95}$ (E) 10
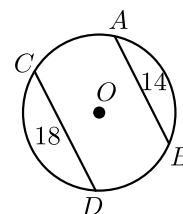
26. An 18 inch diameter pizza is cut into 12 slices. What is the distance around one slice?
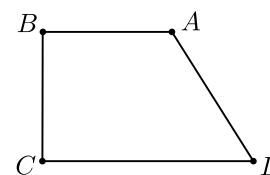
(A) $18 + \frac{\pi}{2}$ (B) $18 + 9\pi$ (C) $\boxed{18 + \frac{3\pi}{2}}$ (D) $18 + \frac{9\pi}{2}$ (E) $36 + \frac{9\pi}{2}$

27. A circular pool with a radius of 10 has a circular walkway constructed around it with a width of 1. Find the area of the walkway.

(A) $\boxed{21\pi}$ (B) 32π (C) 36π (D) 44π (E) 52π

28. In trapezoid $ABCD$ shown, $AB = BC$, $AC = 15\sqrt{2}$ and $AD = 17$. Find the area of the trapezoid.

(A) 180 u^2 (B) 195 u^2 (C) 205 u^2 (D) 250 u^2 (E) $\boxed{285 \text{ u}^2}$
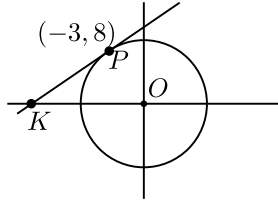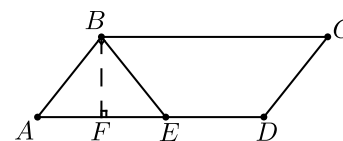

29. The surface area of a box is 6400 m^2 with rectangular sides and a square base. Find the volume of the box if its height is $70 \ m$.

(A) $12,000 \text{ m}^3$ (B) $16,000 \text{ m}^3$ (C) $18,000 \text{ m}^3$ (D) $22,000 \text{ m}^3$ (E) $\boxed{28,000 \text{ m}^3}$

30. In the given parallelogram $ABCD$, $AB = BE = ED = 1$ and $\angle ABE = 90°$. Find the ratio of the area of the trapezoid $BEDC$ to the area of the triangle ABE.

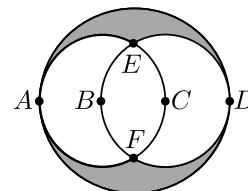(A) $\sqrt{2} + 2$ (B) $\boxed{\sqrt{2} + 1}$ (C) $\sqrt{2}$ (D) $\sqrt{2} - 1$ (E) $\sqrt{2} - 2$

31. What is the area of $\triangle ABC$ with vertices $A(1,3)$, $B(1,-5)$ and $C(7,-8)$?

(**A**) 12 u^2 (**B**) 20 u^2 (**C**) $\boxed{24 \text{ u}^2}$ (**D**) 36 u^2 (**E**) 48 u^2

32. Two complementary angles have measures $6x - 20$ and $4x$. Find the supplement of the smaller angle.

(**A**) $155°$ (**B**) $146°$ (**C**) $142°$ (**D**) $\boxed{136°}$ (**E**) $128°$

33. If the radius of a sphere is halved, what is the ratio of the new volume to the old?

(**A**) $\boxed{1:8}$ (**B**) $1:4$ (**C**) $1:6$ (**D**) $1:2$ (**E**) $1:16$

34. Two chords $\overline{AB}$ and $\overline{DC}$ intersect each other at point O so that $AO = 1.2$, $OB = 7.5$ and $OC = DO$. How long is $\overline{DC}$?

(**A**) $\sqrt{35}$ (**B**) 4.5 (**C**) $\boxed{6}$ (**D**) $\sqrt{8.7}$ (**E**) 9

35. In the figure shown, a line is tangent to the circle at the origin. The point of tangency is $(-3,8)$. The line intersects the x-axis at $x = k$. Find k.

(**A**) $\boxed{-\dfrac{73}{3}}$ (**B**) $-\dfrac{8}{3}$ (**C**) $-\dfrac{64}{3}$ (**D**) $-\dfrac{16}{3}$ (**E**) $-\dfrac{32}{3}$

36. In $\triangle ABC$, segment $\overline{OB}$ bisects angle $\angle ABC$ and $AB = AC$. If $m\angle ABO = 2x + 20$ and $m\angle CBO = 4x + 12$, what is $m\angle BAC$?

(**A**) $96°$ (**B**) $\boxed{68°}$ (**C**) $56°$ (**D**) $38°$ (**E**) $26°$

37. Point (x,y) is 8 units from the origin on the line $y = -x$. Find $|xy|$

(**A**) 64 (**B**) 48 (**C**) $\boxed{32}$ (**D**) 16 (**E**) 8

38. The length of a rectangle is diminished by 10 feet, while the width is increased by 6 feet. If the result is a square whose area is equal to that of the original rectangle, what is the area?

(**A**) 196 feet^2 (**B**) 256 feet^2 (**C**) 169 feet^2 (**D**) $\boxed{225 \text{ feet}^2}$ (**E**) 349 feet^2

39. Find the length of the perimeter of a right triangle whose area is 120 cm^2 and whose hypotenuse is 26 cm.

(**A**) $\boxed{60 \text{ cm}}$ (**B**) 40 cm (**C**) 50 cm (**D**) 70 cm (**E**) 80 cm

40. Jack and Rose each order a pizza. The circumference of Jack's pizza is 30% greater than the circumference of Rose's pizza. The area of Jack's pizza is what percentage greater than the area of Rose's?

(**A**) 30% (**B**) 49% (**C**) 50% (**D**) $\boxed{69\%}$ (**E**) 75%

41. How many different chords are determined by 8 distinct points lying on a circle?

(**A**) $\boxed{28}$ (**B**) 36 (**C**) 31 (**D**) 21 (**E**) 16

42. A sphere of radius r is cut by a horizontal plane at a distance k from the center of the sphere. Find the area of the cross section.

(**A**) $\pi r^2 + \pi k^2$ (**B**) $2\pi k^2 + 2\pi r^2$ (**C**) $\boxed{\pi r^2 - \pi k^2}$ (**D**) $2\pi r^2 - 2\pi k^2$ (**E**) $\pi k^2 - \pi r^2$

43. In the figure shown, two small circles are inscribed and tangent to the larger circle at points A and D. Let points B and C be the centers of the circles, where $AB = BC = CD = 1$. Find the area of the shaded region.

(**A**) $\dfrac{5\pi}{12} - \dfrac{\sqrt{3}}{2}$ (**B**) $\dfrac{13\pi}{24} - \dfrac{\sqrt{3}}{2}$ (**C**) $\dfrac{7\pi}{12} - \dfrac{\sqrt{3}}{2}$

(**D**) $\dfrac{11\pi}{24} - \dfrac{\sqrt{3}}{2}$ (**E**) $\boxed{\dfrac{11\pi}{12} - \dfrac{\sqrt{3}}{2}}$

44. Given the triangle with vertices $A(1,4)$, $B(5,2)$ and $C(9,3)$, determine the distance from the centroid of the triangle to the vertex $A(1,4)$.

(**A**) 4   (**B**) $\sqrt{15}$ (**C**) 5   (**D**) $\boxed{\sqrt{17}}$ (**E**) 3

45. Segment $\overline{AB}$ is a diameter of circle O. P is a point on the circumference such that $AP = 7$ and $BP = 6$. What is the area of circle O?

(**A**) $\dfrac{21\pi}{2}$ (**B**) $\boxed{\dfrac{85\pi}{4}}$ (**C**) 42π (**D**) 85π (**E**) $\dfrac{42\pi}{3}$

46. Given two triangles, which postulate could not be used to show the triangles are similar?

(**A**) SSS   (**B**) AAS   (**C**) SAS   (**D**) $\boxed{\text{SSA}}$ (**E**) ASA

47. In the rhombus $ABCD$, diagonal DB is 20 units longer than diagonal AC. If the area of the rhombus is 150 u^2, what is the length of the longest diagonal?

(**A**) $\boxed{\text{30 u}}$ (**B**) 10 u   (**C**) 50 u   (**D**) 35 u   (**E**) 15 u

48. There are four regular triangular pyramids lined up in a row. The first three pyramids have side lengths of 6, 8 and 10. The volume of the last pyramid is the sum of the volumes of the first three. Determine the side length of the fourth pyramid.

(**A**) 18   (**B**) 4   (**C**) 16   (**D**) 8   (**E**) $\boxed{12}$

49. A triangle has its vertices at points $A(-3,-2)$, $B(-4,1)$ and $C(1,6)$. Which of the following is an equation in slope-intercept form of a line perpendicular to the line that contains the median to side $\overline{BC}$?

(**A**) $y = -3x - \frac{17}{3}$ (**B**) $\boxed{y = -\frac{3}{11}x - \frac{25}{11}}$ (**C**) $y = \frac{11}{3}x + \frac{17}{3}$ (**D**) $y = \frac{9}{11}x + \frac{25}{11}$ (**E**) $y = \frac{11}{9}x + \frac{25}{11}$

50. The measure of each interior angle of a regular polygon is eighteen times that of an exterior angle of the polygon. How many sides does the polygon have?

(**A**) 24   (**B**) 32   (**C**) $\boxed{38}$ (**D**) 42   (**E**) 46